Teaching K-8 April 2005 Math Column Michael Naylor

Math Magic

Surprising Tricks that Build Number Sense and Algebraic Reasoning

Mathematics truly is magical, especially for students with strong number sense and algebra skills! Here's a variety of mathematical surprises that will capture your students' interest and motivate exploration of mathematical ideas. While the tricks themselves are fascinating, push your students to think about the reasons why these stunning feats work – each one is built on a mathematical idea even more amazing than the trick itself!

Math Dice (K-2)

On standard six-sided dice, opposite numbers always add to 7. This forms the basis for several tricks, and can help your students learn the fact family for 7.

Have your students each roll a six-sided die and write down the number they have rolled so they'll remember it. Then ask them to flip their dice over so the number that was on the bottom is now on top. They should now add the new number to their first number. After a dramatic pause, "guess" that the total they now have is 7.

Allow them to try again, and then ask the students for the pairs of numbers they got. You'll generate a list of the additive fact family for 7: 1+6 = 7, 2+5 = 7, etc.

Now have them roll their dice and just by looking at the top number, have them predict which number is on the bottom of their dice.

"Magical" Calculations (3-5)

These mental math tricks are so slick, they seem like magic.

<u>Multiplying by 5:</u> Divide the number by 2 and multiply by 10. For example, to multiply 18 x 5, divide 18 by 2 to get 9, then multiply 9 by 10 to get 90. Give your students a series of numbers to multiply by 5, most of your students will be able to do this quickly in their heads. Wow!

What about odd numbers that don't divide evenly by 2? Just think in terms of decimals. For example, 41 x 5. First take half of 41, which is 20.5, then multiply by 10 to get 205. It works every time.

<u>Multiplying by 11</u>: To multiply a 2 digit number by 11, write the two digits with a space between them, and in that space write <u>the sum</u> of the two digits. For example, to multiply 52 by 11, write 5 __ 2. What goes in the blank? 5+2, or 7, so the answer is 5 <u>7</u> 2.

Give a few more examples, and have your students try them and check with a calculator or pencil and paper to verify the results. Make sure in your examples that the two digits when added together are 9 or less, because it's a little more complicated when they sum to 10 or greater.

Can your students explain why this works? Having them calculate with pencil and paper can help explain.

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     5 2
   x 1 1
   ---------
     5 2
+  5 2
   ---------
   5 7 2
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In the calculation show above, 52 times 11 is *52 tens* plus *52 ones.* Be sure to emphasize the place value concepts in the discussion.

Now, as a problem-solving extension, ask your students to find a way to multiply a two-digit number by 11 when the digits add up to 10 or greater. For example, how would one multiply 64 by 11 using a similar method?

This time, instead of writing a "10" between the six and the four, you write the last digit of "10", or "0" between the 6 and the 4 and then add 1 to the six. What is the place-value explanation? Again, examining the pencil and paper calculation can be very helpful. A "10" in the tens place is the same as 1 hundred.

Mystery Number (4-8)

Read the following instructions to your students:

1. Choose a number from 1 to 20
2. Double it
3. Add 6
4. Divide by 2
5. Subtract the number you originally started with

Pause for dramatic effect, and announce: "The number you now have is 3!" Allow your students to try the trick again with different starting numbers.

An explanation: Draw a box on the board to represent the starting number. We are then asked to double that number, so write "x 2" after the box. Now we add 6, so write a big "6" on the board also. The board will look like this:

[ ] x 2   6

Now we divide everything by 2. Cross out the "6" and write "3", and ask what happens to the "[ ] x 2". Since that part stands for twice the number, when you divide it by two you "undo" the doubling and end up with just the starting number. Cross out the "x 2" part so the board now shows "[ ] ~~x 2~~ ~~6~~ 3". At this point, we now have just the original number plus 3. What happens when we subtract the original number? Only the 3 is remaining – and it doesn't matter what the original was!

Ask your students to try to create their own instructions to predict a mystery number. Remind them that at some point in the calculations they need to subtract the original number.

This problem builds towards some rather sophisticated algebraic reasoning.

Magic Matrix (5-8)

Draw the following grid on the blackboard and ask your students to copy it on paper.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Ask them to circle any number on the grid, and cross out all other numbers in the same row and column as that number. For example, if they chose 10, they would cross out 2, 6, and 14 in the same column, and 9, 11, 12 in the same row.

1	~~2~~	3	4
5	~~6~~	7	8
~~9~~	10	~~11~~	~~12~~
13	~~14~~	15	16

Have them choose a remaining number (not crossed out) and do the same thing: circle it and cross out all the other numbers in the same row and column. Have them do this a third and a fourth time, there will now be four circled numbers and the rest will all be crossed out.

~~1~~	~~2~~	3	~~4~~
~~5~~	~~6~~	~~7~~	8
~~9~~	10	~~11~~	~~12~~
13	~~14~~	~~15~~	~~16~~

Announce the total: 34. It will be 34 every time!
How does it work? Rewrite the grid like this:

	+1	+2	+3	+4
+0	1	2	3	4
+4	5	6	7	8
+8	9	10	11	12
+12	13	14	15	16

The outside numbers are the "generators" of the grid. Each number in the grid is the sum of the two generators in its row and column. For example, 15 is in the +12 row and the +3 column, and 15 = 12 + 3. When numbers are chosen for the trick, there is exactly one number from each column and one number from each row. The total of those numbers, then, is the total of all of the generators, or 1+2+3+4 + 0+4+8+12 = 34. In the example above, the numbers were 3, 8, 10 and 13, which is (0+3), (4+4), (8+2), and (12+1)... the sum of all of the generators!

Here's another grid made with different generators. If the same trick is done on this grid, the total of the four numbers will be 39 every time. Your students can make their own grids with their own generators. Note that grids will also work if the generators are negative numbers or fractions!

	+0	+3	+7	+6
+1	1	4	8	7
+4	4	7	11	10
+10	10	13	17	16
+8	8	11	15	14

Missing Digit (5-8)

To check if a number is divisible by 9, you can add the digits together and see if the sum is divisible by 9. For example, the number 5176 is divisible by 9 because 5+1+7+6 = 18, and 18 is divisible by 9. Here's a wonderful magic trick that works on that principle. Give your students the following directions:

1. Pick a five digit number and multiply it by 9 on a calculator.
2. Write down the resulting 5 or 6 digit number.
3. Choose any digit, <u>except for a zero</u>, and circle it.
4. Find the sum the remaining digits in their number.

Now ask them for their sums. Since they multiplied by 9, the sum of all of the digits in step 2 should be a multiple of 9, either 9, 18, 27, 36, or 45. The sum they tell you will be missing a part – the circled digit. It is easy to tell what that digit is by asking what you need to add to their sum to make it a multiple of 9.

For example, suppose a student tells you "14." Their circled digit must be "4", since 14 + 4 = 18. If another student tells you their sum is 30, they must have circled a 6, since 30 + 6 = 36.

Your students will be puzzled and amazed by this trick. After making several predictions, start writing students' sum and circled numbers on the board. After several examples, your students will begin to see the pattern: the total of the sum and the circled number always adds to a multiple of 9.

This trick is especially powerful if you've already been studying the divisibility test for 9.